THE SCALE OF NON-ADIABATIC HEATING AS A FACTOR IN CYCLOGENESIS

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ABSTRACT

With the aid of a simple linearized model, the effect of large-scale atmospheric heat sources on a resting atmosphere has been investigated. Particular emphasis has been devoted to the study of the dependence of pressure change on the horizontal dimension of the heat sources.

It is thus found that, when the maximum intensity in the center of the heat source is independent of the horizontal dimension, the pressure fall at the surface has a maximum when the width is about 4000 km in the case of no friction. With friction included, the pressure change also depends on the rate of heating. With decreasing rate of heating, the pressure fall at the surface thus decreases while the width for which the maximum occurs increases (4000 to 5000 km). The mean wind decreases monotonically with increasing width of the heat source except for the very smallest scales.

In the case where the maximum intensity in the center of the heat source is proportional to the horizontal extent, the pressure fall at the surface increases monotonically with increasing dimensions.

The dependence on the latitude and the stability is investigated.

1. Introduction

During the last years, it has several times been very clearly demonstrated that the heat sources and sinks of the earth are important factors in the generation of weather systems. A number of investigations have dealt with the modification of air masses moving over these heat sources and sinks. Burke (1945) and Klein (1946) studied the transformation of polar-continental to polar-maritime air at the east coast of North America. Craddock (1951) studied the warming of air masses over eastern North Atlantic, and Burbridge (1951) made a study of the modification of continental polar air over Hudson Bay. Results from these investigations show that the change of the surface temperature could exceed 20C in 24 hr and affect deep layers of air.

The dynamic influence of the non-adiabatic heating has been studied by several authors. Petterssen (1950) demonstrated that, in the winter, there is a marked tendency for new cyclones to form and existing ones to intensify over water bodies surrounded by colder land. He also showed that, in summer, anticyclones develop over water bodies surrounded by warmer land. Another investigation which demonstrated that the heat sources can be of importance in the formation of cyclones has been undertaken by Winston (1955). He studied the rapid development of large-scale cyclonic activity which often takes place in the Gulf of Alaska. The effect of barotropic redistribution of vorticity seems to account for much of the development. However, it is found that there exist pronounced fields of divergence and vertical motion which, particularly at the time of the most rapid development, reflect the influence of heat sources on the circulation. Petterssen (1955) points out that it is not the amount of heating and cooling itself, but rather the configuration of the patterns, which is important. To be effective, the heat sources and sinks must be scaled to the motion systems.

Smagorinsky (1953) investigated, with the aid of a thermally active model, how the large-scale heat sources and sinks of the atmosphere produce quasi-static and quasi-geostrophic perturbations.

In the present investigation, we will mainly be concerned with the effect of the scale of the heat sources. The model we consider is very simple but may prove to be sufficiently realistic for this limited purpose.

2. The model

We consider a column of air in which there is no motion in the initial state. Heating in the lower part gives rise to a flow along the surface towards the central part of the heated area. This inward flow will decrease to zero at a certain level, and above that level there will be outward flow. Because of the Coriolis force, the inflowing current will develop

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cyclonic rotation and the outflowing current anticyclonic rotation. A qualitative discussion of such a thermally produced motion is given, for example, by Godske *et al.* (1957).

Regarding the effect of friction, we will disregard this in the free atmosphere and only take it into account in the friction layer in the way that we use the vertical velocity produced by friction on top of the friction layer as the lower boundary condition. The lower boundary ($z = 0$) is therefore moved up to the top of the friction layer. In this connection, it is to be mentioned that the classical concept of a friction layer over the sea has been criticized by Sheppard, Charnock and Francis (1952). They found from observations that there is no substantial decrease of the stress with height in the first few hundred metres as required by the friction-layer theory. However, Lettau (1957) has shown in a more detailed study of the same observation material that there is no basic difference between the turbulence and shearing-stress conditions over the sea compared to those over land.

The horizontal variation of the Coriolis parameter is disregarded. This approximation certainly makes the model less realistic but is not likely to be too severe in this connection.

The basic equations we have at our disposal to study this thermally-produced circulation are the equations of vorticity and continuity, the first law of thermodynamics, the equation of state and the hydrostatic equation:

$$\frac{d}{dt}(\zeta + f) = -(\zeta + f)\nabla\cdot\boldsymbol{v} + \frac{1}{\rho^2}\boldsymbol{k}\cdot\nabla\rho \times \nabla p + \boldsymbol{k}\cdot\frac{\partial \boldsymbol{v}}{\partial z} \times \nabla w, \quad (1)$$

$$\frac{d\rho}{dt} + \rho\nabla\cdot\boldsymbol{v} + \rho\frac{\partial w}{\partial z} = 0, \quad (2)$$

$$q = c_p\frac{dT}{dt} - \frac{1}{\rho}\frac{dp}{dt}, \quad (3)$$

$$\rho = \frac{p}{RT}, \text{ and} \quad (4)$$

$$\frac{\partial p}{\partial z} = -g\rho, \quad (5)$$

where ζ is the relative vorticity, $\boldsymbol{v}$ the horizontal velocity vector, w the vertical velocity component, f the Coriolis parameter, ∇ the horizontal gradient operator, $\boldsymbol{k}$ the vertical unit vector, q the rate of heat gained or lost per unit mass and unit time, ρ the density, p the pressure, T the temperature, R the gas constant, g the acceleration of gravity and c_p the specific heat for air at constant pressure.

We now consider perturbed motion and introduce the following new variables:

$$\boldsymbol{v} = \boldsymbol{v}',$$
$$w = w',$$
$$p = p_0 + p',$$
$$\rho = \rho_0 + \rho', \text{ and}$$
$$T = T_0 + T'.$$

The variables with the subscript zero represent the undisturbed state where the isobaric, the isopycnic and the isothermal surfaces are horizontal; the atmosphere is in hydrostatic equilibrium—

$$\frac{\partial p_0}{\partial z} = -g\rho_0. \quad (6)$$

The undisturbed temperature is assumed to decrease linearly with height,

$$T_0 = T_0(0) - \gamma z, \quad (7)$$

where $T_0(0)$ is the temperature at the top of the friction layer and γ is the lapse rate.

With the aid of these assumptions, and by elimination among the linearized basic eq (1)–(5), the following partial differential equation is obtained:

$$\frac{\partial^2}{\partial z^2}\frac{\partial}{\partial t}\left(\frac{p'}{\rho_0}\right) - \frac{g}{RT_0}\frac{\partial}{\partial z}\frac{\partial}{\partial t}\left(\frac{p'}{\rho_0}\right) + \frac{g(\gamma_d - \gamma)}{fT_0}\frac{\partial \zeta'}{\partial t} = \gamma_d\frac{\partial}{\partial z}\left(\frac{q}{T_0}\right) - \frac{g\gamma_d}{RT_0^2}q, \quad (8)$$

where γ_d is the dry adiabatic lapse rate. We shall now make the assumption that the heating of the air is so slow that at each instant the motion induced will be approximately in dynamic equilibrium. Then it is possible to make use of the geostrophic-wind equation. We thus assume

$$\boldsymbol{v}' = -\frac{1}{\rho_0 f}\nabla p' \times \boldsymbol{k}. \quad (9)$$

This gives, for the vorticity,

$$\zeta' = \frac{1}{\rho_0 f}\nabla^2 p'. \quad (10)$$

Introducing this in eq (8) gives the following second-order partial differential equation in $\partial p'/\partial t$:

$$\left[\frac{\partial^2}{\partial z^2} - \frac{g}{RT_0}\frac{\partial}{\partial z} + \frac{g(\gamma_d - \gamma)}{f^2 T_0}\nabla^2\right]\frac{1}{\rho_0}\frac{\partial p'}{\partial t} = \gamma_d\frac{\partial}{\partial z}\left(\frac{q}{T_0}\right) - \frac{g\gamma_d}{RT_0^2}q, \quad (11)$$

which is of the elliptic type (provided $\gamma_d \geqslant \gamma$). This equation may be compared with an equation Charney

(1949) derived in connection with a study of filtering methods to eliminate small-scale noise. In his derivation, the equations were not linearized and the motion was assumed to be adiabatic. In this case, the right side of (11) contains the (geostrophic) advections of temperature and vorticity in place of the non-adiabatic term in q. The left side of (11), however, is the same.

We will solve this equation under certain simplified conditions. Regarding the non-adiabatic heating, we shall assume it has the following distribution:

$$q = Q(0)e^{-z/h} \sin kx \sin ky, \tag{12}$$

where $Q(0)$ is the heat gain per unit mass and unit time at the level $z = 0$ in the center of the column, h is the height at which it has decreased to $1/e$ of this value, and $k = \pi/D$, D is the width of the column.

Consistent with this variation of the non-adiabatic heating, it is natural to assume that the horizontal variation of the pressure change can be represented by

$$\frac{\partial p'}{\partial t} = \frac{\partial P}{\partial t} \sin kx \sin ky. \tag{13}$$

Since the factor $1/T_0$ in the coefficients of eq (11) varies so slowly with height, we may disregard this variation. We therefore introduce the following constants:

$$\begin{cases} H = \dfrac{RT_m}{g}, \quad \text{and} & (14) \\[2ex] s = g\dfrac{\gamma_d - \gamma}{T_m}, & (15) \end{cases}$$

where T_m is the mean temperature with respect to height, and s is a measure of the static stability. We further neglect the vertical variation of γ.

After making use of eq (12) through (15), eq (11) becomes

$$\left[\frac{\partial^2}{\partial z^2} - \frac{1}{H}\frac{\partial}{\partial z} - 2\frac{sk^2}{f^2}\right]\frac{1}{\rho_0}\frac{\partial P}{\partial t} = -Q(0)\frac{R}{c_pH}\left(\frac{1}{h} + \frac{1}{H} - \frac{\gamma}{T_m}\right)e^{-z/h}. \tag{16}$$

The solution to this differential equation for the pressure tendency is

$$\frac{1}{\rho_0}\frac{\partial P}{\partial t} = A(t)e^{(1+k_1)z/2H} + B(t)e^{(1-k_1)z/2H} - \alpha Q(0)e^{-z/h}, \tag{17}$$

where

$$k_1 = \left(1 + \frac{8H^2sk^2}{f^2}\right)^{\frac{1}{2}} \tag{18}$$

and

$$\alpha = \frac{\dfrac{R}{c_pH}\left(\dfrac{1}{h} + \dfrac{1}{H} - \dfrac{\gamma}{T_m}\right)}{\dfrac{1}{h}\left(\dfrac{1}{h} + \dfrac{1}{H}\right) - \dfrac{2sk^2}{f^2}}. \tag{19}$$

$A(t)$ and $B(t)$ are arbitrary functions of time which have to be determined from the upper and lower boundary conditions. It is obvious the local pressure change must remain finite at great heights. Thus, $A(t)$ must be zero. Before determining $B(t)$ from the lower boundary condition, we integrate (17) with respect to time. In the initial state, we have assumed there is no motion. By disregarding the variation in time of h, γ and T_m and by defining

$$\beta(t) = \int_0^t B(t)dt, \tag{20}$$

we can write

$$\frac{P}{\rho_0} = \beta(t)e^{(1-k_1)z/2H} - \alpha e^{-z/h}\int_0^t Q(0)dt. \tag{21}$$

To determine $B(t)$, we proceed in the following indirect way. By combining (1), (2), (4) and (6), we obtain after linearization the following equation:

$$\frac{\partial}{\partial z}\left(\frac{\partial p'}{\partial t} - g\rho_0 w'\right) + \frac{g\rho_0}{f}\frac{\partial \zeta'}{\partial t} = 0. \tag{22}$$

Integration from $z = 0$ to infinity (where we require the individual change of pressure to be zero) gives

$$\left(\frac{\partial p'}{\partial t} - g\rho_0 w'\right)_{z=0} = \frac{g}{f}\int_0^\infty \rho_0 \frac{\partial \zeta'}{\partial t}dz. \tag{23}$$

Regarding the vertical velocity at $Z = 0$, we assume that this is mainly due to ground friction. An expression for this contribution has been derived by Charney and Eliassen (1949):

$$w'(0) = \left(\frac{2K}{f}\right)^{\frac{1}{2}} \sin \nu \cos \nu \cdot \zeta_g, \tag{24}$$

where K is the eddy viscosity, ν the angle between the surface wind and surface isobars and ζ_g the geostrophic vorticity. Introducing this expression for the vertical velocity (24) in (23) and making use of (10) and (13) gives

$$\left[\frac{\partial P}{\partial t} + \frac{gk^2}{f}\left(\frac{2K}{f}\right)^{\frac{1}{2}} \sin 2\nu \cdot P\right]_{z=0} = -2g\frac{k^2}{f^2}\int_0^\infty \frac{\partial P}{\partial t}dz. \tag{25}$$

By introducing the solution (21) here and by assuming the vertical variation of the density to be

$$\rho_0(z) = \rho_0(0)e^{-z/H^*}, \tag{26}$$

where H^* is the height of the homogeneous atmosphere, we obtain the following ordinary differential equation for $B(t)$:

$$\frac{dB}{dt} + \lambda_2 B = \alpha\left[\lambda_1 Q(0) + \lambda_2 \int_0^t Q(0)dt\right], \quad (27)$$

where

$$\lambda_1 = \frac{1 + 2g\dfrac{k^2}{f^2}\dfrac{hH^*}{h + H^*}}{1 + 4g\dfrac{k^2}{f^2}\dfrac{HH^*}{H^*(k_1 - 1) + 2H}} \quad (28)$$

$$\lambda_2 = \frac{\dfrac{gk^2}{f}\left(\dfrac{2K}{f}\right)^{\frac{1}{2}}\sin 2\nu}{1 + 4g\dfrac{k^2}{f^2}\dfrac{HH^*}{H^*(k_1 - 1) + 2H}}. \quad (29)$$

In order to solve this equation, it is necessary to specify the time variation of the heating. We chose the following simple case:

$$Q(0) = W = \text{constant}. \quad (30)$$

Eq (27) can then be written

$$\frac{d\beta}{dt} + \lambda_2\beta = \alpha W(\lambda_1 + \lambda_2 t). \quad (31)$$

With the boundary condition $\beta(0) = 0$, the solution to (31) becomes

$$\beta = \alpha Wt\left[1 - (1 - \lambda_1)\frac{1 - e^{-\lambda_2 t}}{\lambda_2 t}\right]. \quad (32)$$

The solution to the differential eq (11) can now be written

$$p'(z) = \rho_0(0)\alpha Wt\left\{\left[1 - (1 - \lambda_1)\frac{1 - e^{-\lambda_2 t}}{\lambda_2 t}\right] \times \exp\left(\frac{1 - k_1}{2H} - \frac{1}{H^*}\right)z - \exp\left[-\left(\frac{1}{h} + \frac{1}{H^*}\right)z\right]\right\}\sin kx \sin ky, \quad (33)$$

where the expressions for k_1, α, λ_1 and λ_2 are given in (18), (19), (28) and (29) respectively. In case we disregard the effect of friction, solution (33) reduces to

$$p'(z) = \rho_0(0)\alpha Wt\left\{\lambda_1 \exp\left(\frac{1 - k_1}{2H} - \frac{1}{H^*}\right)z - \exp\left[-\left(\frac{1}{h} + \frac{1}{H^*}\right)z\right]\right\}\sin kx \sin ky. \quad (34)$$

3. Results

In studying the behavior of the solution, we will use the following numerical values for the parameters:

$$\begin{aligned}
Q(0) &= W = 10^8 \text{ erg gm}^{-1}\text{ day}^{-1}\\
\rho_0(0) &= 1.2\cdot 10^{-3}\text{ gm cm}^{-3}\\
T_m &= 256°\text{K}\\
\gamma &= 6.5\cdot 10^{-5}\ °\text{K cm}^{-1}\\
s &= 1.2\cdot 10^{-4}\text{ sec}^{-2}\\
H &= 7.5\cdot 10^5\text{ cm}\\
h &= 3.75\cdot 10^5\text{ cm}\\
H^* &= 8\cdot 10^5\text{ cm}\\
K &= 0.5\cdot 10^5\text{ cm}^2\text{ sec}^{-1}\\
\nu &= 20°\\
f &= 10^{-4}\text{ sec}^{-1}
\end{aligned}$$

The total amount of heat added to the atmosphere per unit time above the friction layer (in the center of the column) is obtained by integrating with respect to z:

$$Q_1 = \int_0^\infty \rho_0 Q(0)e^{-z/h}dz. \quad (35)$$

With the aid of the density distribution (26), we then have

$$\begin{aligned}
Q_1 &= \frac{\rho_0(0)Q(0)}{\dfrac{1}{h} + \dfrac{1}{H^*}}\\
&= 3.06\cdot 10^{10}\text{ erg cm}^{-2}\text{ day}^{-1} \quad (36)\\
&= 730\text{ cal cm}^{-2}\text{ day}^{-1}.
\end{aligned}$$

According to Jacobs (1949), the total amount of heat made available in form of sensible heat (Q_c) and through condensation and precipitation of water vapor (Q_p) at the east coasts of North America and Asia is during the winter in the mean more than 350 cal cm^{-2} day^{-1}. For individual days, it could, of course, be much more. He also found that the main contribution is in the form of sensible heat ($Q_c > 2Q_p$). The heating due to radiation (Q_r) is of less interest in this connection. It has been pointed out (for example, by Mintz, 1955), that the magnitude of the radiation effect is high relative to the other heating effects but that the variance probably is one order of magnitude smaller.

It is obvious that the accuracy of the solution becomes less good in cases where the heat flux is so strong that it leads to moderate or strong convective activity.

With this in mind, the value chosen for the rate of heating seems to be reasonable. As mentioned above, the greater part of the heating is due to sensible heat gain from the ocean surface. Regarding the vertical distribution of the heating, Winston (1955) found that

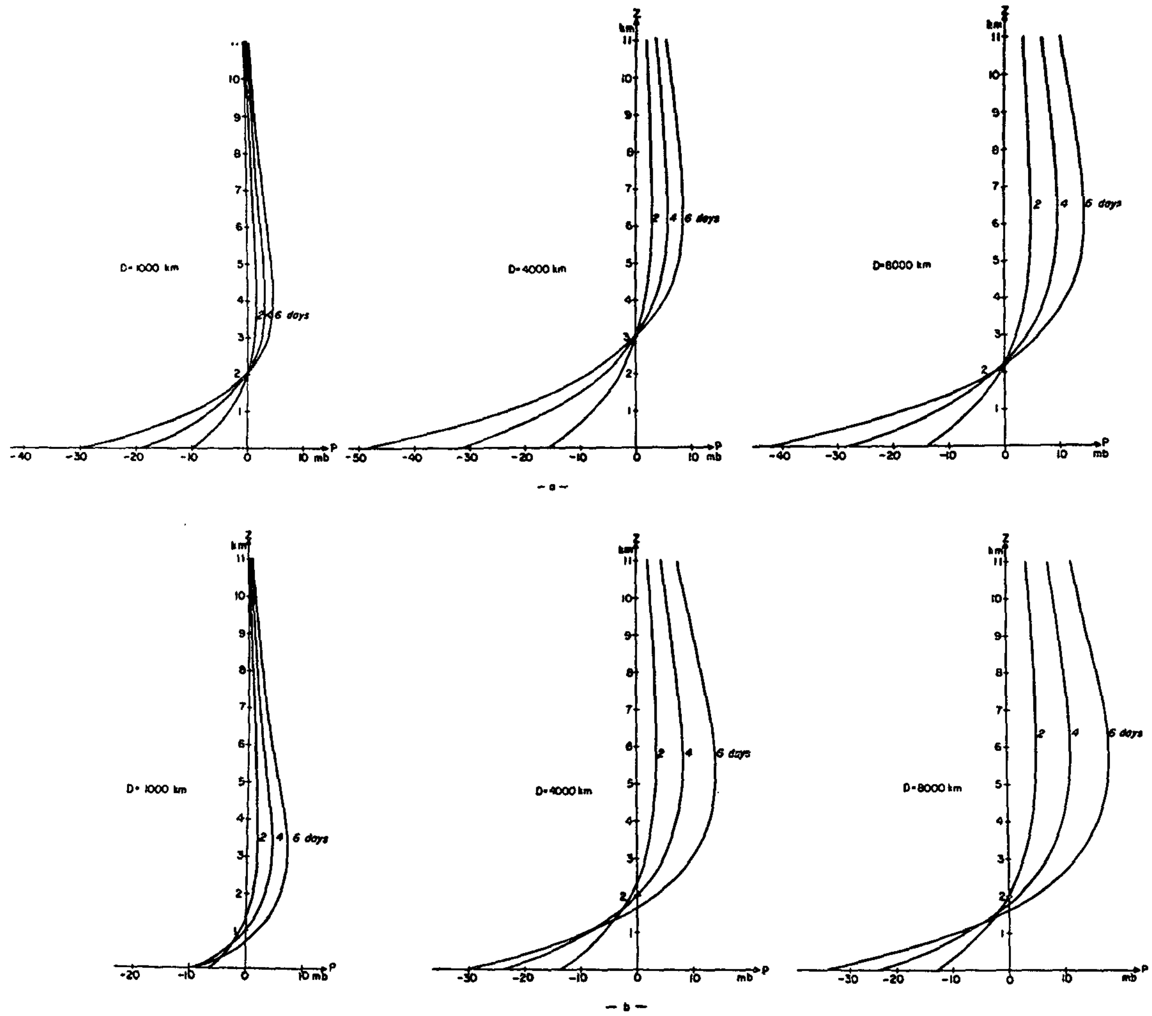

FIG. 1. The vertical variation of the pressure change in the center of the column for various widths of the heat source after t = 2, 4 and 6 days. (a) Without friction, and (b) with friction.

the major part of the heating is found in the layer below 700 mb but that the contribution in the layer 700 to 500 mb is by no means negligible. The variation with height of the heating used in the model therefore appears to be realistic.

The method we have made use of to introduce the effect of friction indeed simplifies the problem very much. However, the largest uncertainty does probably not depend on this. Without regard to how friction is introduced, it remains to assign values to some parameters, such as the eddy viscosity K and the cross isobar angle ν which we know with very little accuracy. In the following, we will only discuss the case when the heating function (12) is positive—*i.e.*, when we have a heat source. In case the heating function is negative, the solution for the pressure change reverses sign only.

By using (33) and (34), the variation in the vertical of the pressure change in the center of the column has been evaluated for different values of the width of the heat source (D = 1000, 4000 and 8000 km) and is shown in fig. 1a (without friction) and fig. 1b (with friction). Also, the variation with respect to time is represented in these figures; the pressure change is shown after 2, 4 and 6 days. As expected, the heating thus causes the pressure to fall in a lower layer (cyclonic motion), and above this layer there is a pressure rise (anticyclonic motion).

We have assumed frictional convergence below $Z = 0$ which in turn gives rise to a vertical velocity at $Z = 0$. We therefore must expect the pressure to increase throughout the column due to the friction. The cyclonic motion in the lower layer becomes less intense, and the anticyclonic motion will be stronger when we introduce friction; the height of the level of non-divergence decreases.

With the heating function chosen, the field of motion cannot reach a steady state when time goes to infinity. However, at $Z = 0$ in the case where friction is taken into account, the pressure will tend towards a constant value when $t \to \infty$.

Fig. 1 clearly illustrates that the effect of the

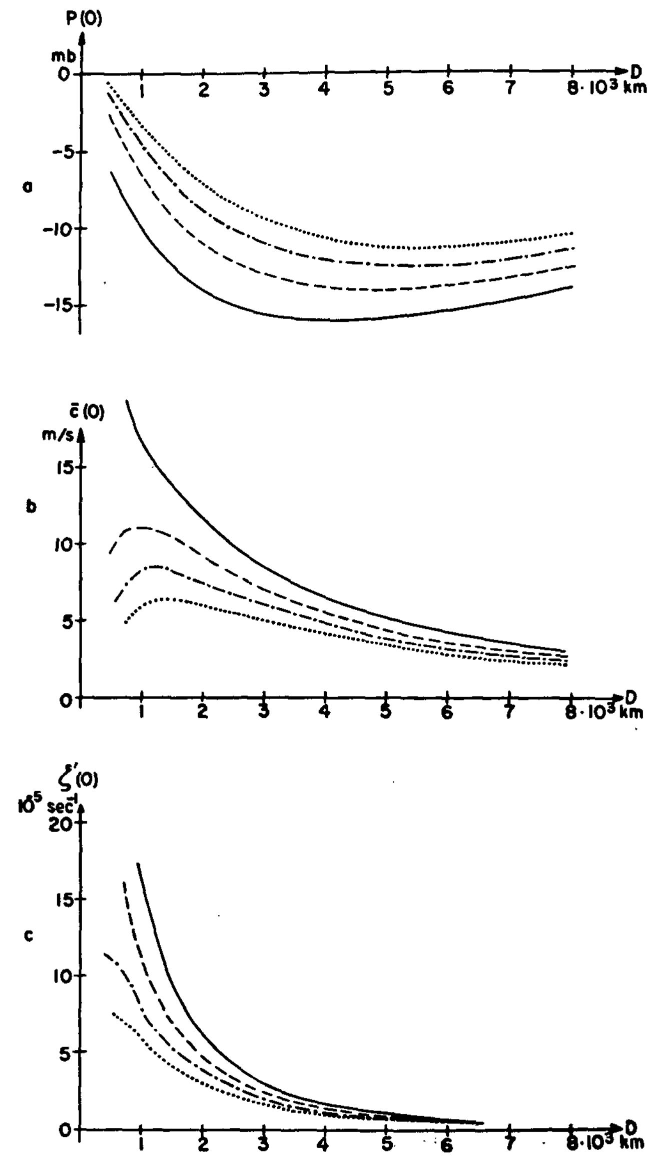

FIG. 2. (a) The pressure change at $Z = 0$ in the center of the column as a function of the width of the heat source. (b) The corresponding mean geostrophic wind. (c) The vorticity. The solid curves represent the case without friction. The other curves are obtained with friction for various values of the rate of heating. The dashed curve: $W = 10^8$ erg gm^{-1} day^{-1}, $t = 2$ days. The dashed-dotted curve: $W = \frac{1}{2}\cdot 10^8$ erg gm^{-1} day^{-1}, $t = 4$ days. The dotted curve: $W = \frac{1}{3}\cdot 10^8$ erg gm^{-1} day^{-1}, $t = 6$ days.

heating is dependent on the scale of the heat source. In order to study this further, the pressure change in the center $P(0)$ has been evaluated as a function of D (fig. 2a). The corresponding mean geostrophic wind defined by

$$\bar{c}(0) = \frac{1}{\rho_0(0)f}\frac{2P(0)}{D} \qquad (37)$$

and the vorticity are shown in figs. 2b and 2c respectively.

In these figures, we can also study the effect of the rate of heating. As can be seen from solution (33), the effect of the friction increases if the amount of heat is given off to the atmosphere during a longer time interval. In case of no friction, as shown by eq (34), the pressure change is simply proportional to Wt.

The four curves in these figures represent four different cases which all have in common that $Wt = 2\cdot 10^8$ erg gm^{-1} is the same (the same as in fig. 1a and 1b for 2 days). The solid curve is obtained without friction taken into account, while the other three curves are obtained with friction for three different values of the rate of heating, 10^8, $\frac{1}{2}\cdot 10^8$ and $\frac{1}{3}\cdot 10^8$ erg gm^{-1} day^{-1}. The time intervals for these three cases are thus 2, 4 and 6 days.

Because of the geostrophic assumption, we cannot draw any conclusions when the size of the system is small, say less than 500 km. With increasing D, the pressure fall at $Z = 0$ increases to a maximum of approximately 16 mb when D is about 4000 km in the case when we disregard the influence of friction. With friction included, the maximum is smaller and occurs for a somewhat higher value of D. We observe that the relative influence of friction decreases with increasing D which is explained by the fact that the increase of mass in the friction layer due to cross isobaric flow, and thereby also the vertical velocity at the top of the friction layer, by and large are inversely proportional to D.

The mean wind $\bar{c}(0)$ decreases monotonically with increasing D except for the very smallest scales where the influence of friction becomes very important.

As can be seen from the expressions for k_1, α, λ_1 and λ_2 the solutions also depend on the latitude and the stability. To illustrate the latitudinal dependence, the pressure change $P(0)$ has been evaluated as a function of D for different values of the latitude φ = 25N, 45N and 65N while the value of the stability is kept unchanged. This is shown in fig. 3a. In order not to make the figure too unclear, these variations are given only for the non-frictional case. The influence of the latitude then occurs in such a way that it is possible to consider $P(0)$ as a function of fD. From this, it follows that the extreme of $P(0)$ is independent of f. It is only the value of D^* for which the minimum occurs that can vary. We observe that D^* increases with decreasing latitude.

The effect of the stability has been isolated by evaluating $P(0)$ for different values of the lapse rate, γ = 4.5C, 6.5C and 8.5C/km at latitude 45N (*cf.* fig. 3b). We notice that with increasing stability the extreme of $P(0)$ decreases and that the value of D^* increases.

So far, we have been concerned with studying the influence of the heating as a function of the horizontal extent of a heat source. The heating in the center has been considered to be constant. The mean horizontal gradient of the heating $(2Q_1/D)$ thus decreases with increasing D. It may therefore be of interest to study the dependence of the scale by using the

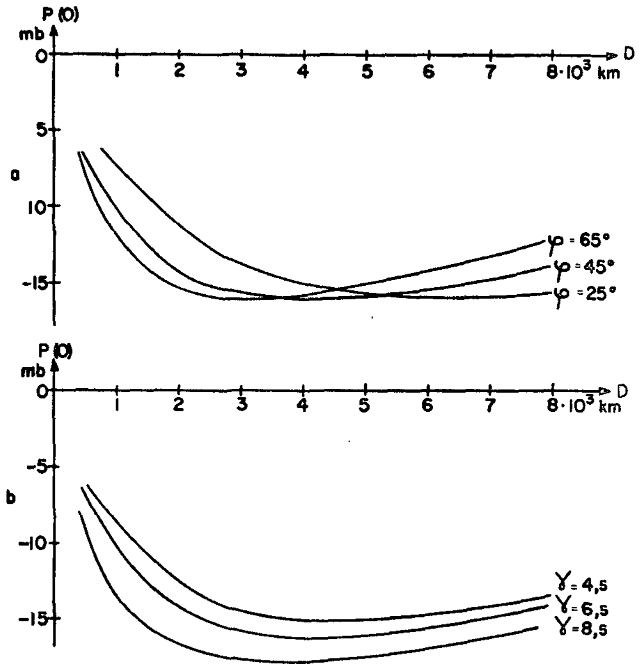

FIG. 3. The pressure change at $Z = 0$ (without friction) in the center of the column as a function of the width of the heat source for different values of (a) the latitude and (b) the lapse rate.

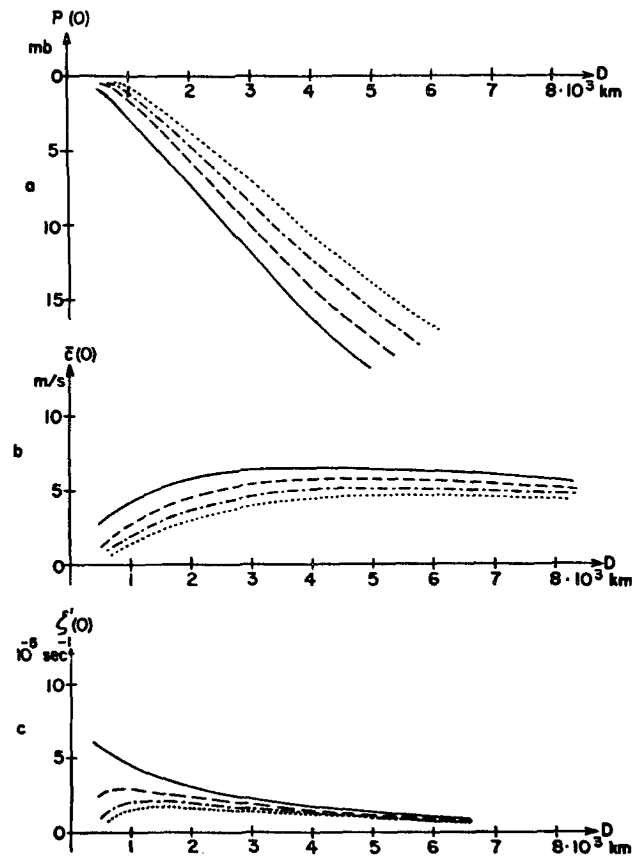

FIG. 4. Same as fig. 2, but now for a heating function (38) where the value in the center is proportional to the horizontal extension of the heat source.

following heating function:

$$q = \frac{D}{D_0} Q(0) e^{-z/h} \sin kx \sin ky, \qquad (38)$$

where $D_0 = 4000$ km. The mean horizontal gradient of the heating then becomes 365 cal cm^{-2} day^{-1} per 1000 km. In this case (*cf.* fig. 4), the pressure change $P(0)$ increases steadily with increasing D while the mean wind $\bar{c}(0)$ now shows a similar variation as the pressure change in the previous case. Thus, the mean wind now has a maximum for about $D = 4000$ km.

4. Comparison with the theory of development

In an extension of Sutcliffe's theory of development, Petterssen (1955; 1956) showed that the direct effect of a heat source on the local change of vorticity at sea level, in the case when the atmosphere is at rest relative to the earth, could be represented by

$$\left(\frac{\partial \zeta}{\partial t}\right)_{z=0} = -\frac{R}{c_p f} \nabla^2 \int_{P_L}^{P_S} q d(\log p), \qquad (39)$$

where the subscripts S and L refer to the sea level and the level of non-divergence, respectively. Applying this formula to the heating and temperature distribution we have assumed gives

$$\left(\frac{\partial \zeta}{\partial t}\right)_{z=0} = \frac{2\pi^2}{c_p f D^2} Q(0) \int_0^{z_L} \frac{e^{-z/h}}{T_0(0) - \gamma Z} dz. \qquad (40)$$

In order to be able to make a comparison with the results we have obtained, we integrate (39) with respect to time. As before, we assume h and γ to be independent of time. This gives then the following expression for the local change of vorticity in the center of the heat source:

$$\zeta(0) = \frac{2\pi^2 g}{c_p f D^2} Wt \int_0^{Z_L} \frac{e^{-z/h}}{T_0(0) - \gamma z}. \qquad (41)$$

This method of studying the effect of non-adiabatic heating is less complete. It only gives the vorticity change at the surface. Furthermore, it is necessary to know the level of non-divergence. This makes it, of course, impossible to compare the two methods directly. In the evaluation of the vorticity at $Z = 0$ as a function of D with the aid of (41), we therefore have to make an estimate for the height of this level. The two upper curves in fig. 5 represent the variation of the vorticity for two different values of Z_L, 3000 m and 5000 m. These two variations of the vorticity at $Z = 0$ with the size of the heat source computed in this way are compared with our results previously shown in fig. 2c and given also here.

From these curves, we can immediately conclude that the level of non-divergence has a relatively strong influence on the result. We also observe that

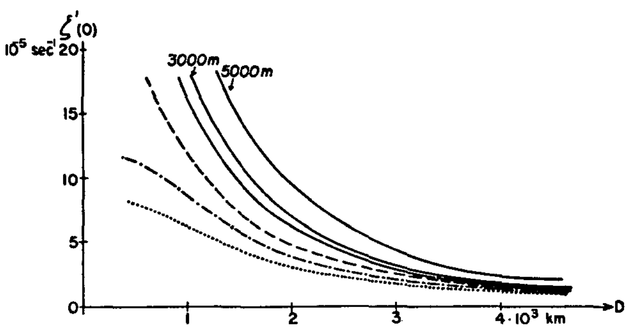

FIG. 5. The vorticity at $Z = 0$ in the center of the column as a function of the width of the heat source obtained with the present model compared with the variation computed from the expression (41) using two different values of the level of non-divergence.

the variation obtained from the theory of development (41) in the case where we used $Z_L = 3000$ m agrees quite well with our case without friction.

5. Remarks

With the aid of this simple model, we have thus obtained certain ideas regarding the dynamic influence on the atmosphere due to non-adiabatic heating. Of particular interest is the effect of the scale of the heat sources as a factor in cyclogenesis.

It is indeed difficult to judge how realistic the model is. An estimate of this is only possible to obtain from comparison with actual situations where we know the horizontal and vertical distribution of the heat exchange. The accuracy with which we can measure the heat transfer is, however, very low. In a comparison with actual situations, it is necessary to isolate the effects which are due to the differential heating. Such methods have been developed and used in an investigation by Petterssen (1959) where he studied the effect of the Great Lakes on the motion and precipitation patterns during a cold spell. He made use of a method developed by Sangster (1959) to calculate the rotational part of the motion. Another method he used consists in resolving the pressure patterns into a set of orthogonal polynomials and has been developed by Wadsworth and Bryan (1948).

We shall also make some comments regarding the assumption that the stability is constant in space and time. This is certainly not true. In the initial state, it may be natural to assume that the stability is constant. However, in the course of time, the stability decreases in the lower part of the column because of the heating. In the upper part, the stability increases since the arrival of potentially cold air from below produces cooling. The fact that the stability varies in space and time violates in turn the assumption that the heating of the column does not change with time. The eddy flux of heat is namely dependent on the stability.

Even more important is perhaps the neglect of temperature and vorticity advection. Since the atmosphere is not at rest initially, and as shown by Smagorinsky (1953), this changes the resulting disturbance very much.

The model thus suffers from a number of more-or-less severe approximations. These are, however, difficult to remove unless we make use of numerical methods to integrate the equations.

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